Mechanics of the Leading Edge

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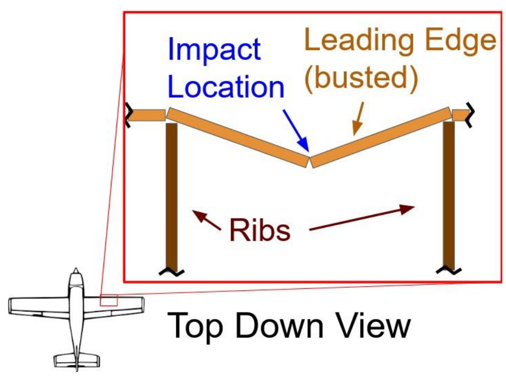
In a previous installment, I did some experiments to find the Young's Modulus and failure stress for balsa as a function of density. Now that we have these material properties in hand, we can use beam bending equations to investigate the quandary that has troubled modelers since the dawn of indoor free flight: “what leading edge won’t break when my plane hits a basketball hoop?”

There are lots of ways a free flight model can break indoors. One common scenario in my personal experience has been that the leading edge busts in between two wing ribs. Let’s look at a section of the leading edge between two wing ribs by thinking about it like a beam fixed by pin joints. The beam corresponds to the leading edge. And the pin joints correspond to where the LE is glued to the front of the wing ribs.



Before it breaks, the leading edge acts like a spring because it deforms more the harder you push on it (the deformation is the distance that the leading edge has been pushed in). For “small” deformations, the deformation is proportional to the force pushing on the LE, and the proportion is called the effective spring constant. We can look up the formula for the effective spring constant of a beam loaded this way in a beam deflection table.

$$P = k_{eff}\delta \qquad k_{eff} = \frac{48EI}{L^3}$$

The equations for deflection of a spring and the effective spring constant for a beam with pin joins at each end and a load in the center. P is the force, k_{eff} is the effective spring constant, δ is the deformation, E is the Young’s Modulus, I is the area moment of inertia, and L is the length (which corresponds to the rib spacing).

The area moment of inertia depends on the cross section dimensions of your LE stick. The formula for area moment of inertia of a rectangle is shown below. Formulas exist for other shapes. I tilt all my square leading edges at 45 degrees to get a sharper LE, and it is of note that the area moment of inertia for a square stick tilted 45 degrees winds up being almost the same as the un-tilted stick.

$$I = \frac{1}{12} hb^3$$

Formula for area moment of inertia, I, of a rectangular leading edge. h is the height of the leading edge, and b is the width of the leading edge.

As the LE deforms, it absorbs energy, but the stick cannot absorb any more energy once the stress within it reaches the failure stress (which we found earlier in part 1 as a material property of the balsa). If the airplane were to crash in a way that forced the stick to absorb any more energy, it would break. The energy absorbed by a spring depends on the spring constant and the deformation.

$$U_{spring} = \frac{1}{2} k_{eff} \delta^2$$

Equation for energy contained in a spring. K is the effective spring constant and δ is the deformation of the spring.

Our goal is to figure out what crash speed imparts that much energy into the leading edge. To find the energy, we need to know the displacement. To calculate the displacement, we need to know the force. Using the equation that relates force and stress within a beam, we can find the force required to create a stress that breaks the stick.

$$P = \frac{4\sigma_{fail} I}{Ly}$$

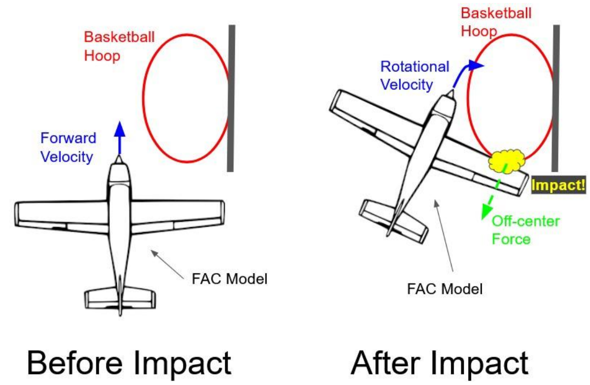
Equation for force (P) required to break the leading edge. σ_{fail} is the failure stress, I is the area moment of inertia, L is the rib spacing (i.e. beam length), and y is half the width of the leading edge.

We can solve for the deformation at the point of failure by dividing the force by the effective spring constant. Plugging that into the energy equation gives the energy required to break the leading edge. This is the same as the energy absorbed by the leading edge at the point of failure.

$$U_{spring} = \frac{1}{2} \left(\frac{I\sigma_{fail}^2 L}{3Ey^2}\right)$$

Equation for energy absorbed by the leading edge at point of failure. I is the area moment of inertia, σ_{fail} is the failure stress, L is the rib spacing (i.e. beam length), E is the Young's Modulus, and y is half the width of the leading edge.

When a model crashes into a basketball hoop, it stops moving forward, so the kinetic energy from the model's motion in flight is converted into other forms of energy. Some of this energy is absorbed by the leading edge, but not all of it. The model usually rotates about the point of contact when it hits a basketball hoop, and this rotation absorbs some of the kinetic energy by turning it into rotational energy.



The rotation is caused because the force stopping the model is exerted a distance away from the center of gravity. The energy absorbed by the rotation depends on the force, the angle of rotation, and where the impact occurs along the wing. Unfortunately, this is a point where some guesswork enters the equation. Based on my memory of indoor ships crashing, the angle of rotation seems often to be about 60 degrees. This value seems to give reasonable final answers, so let's stick with it for now. We derived the formula for breaking force earlier, so let's substitute it into the equation for rotational energy ($U = P \times r \times \theta$). The point of contact with the basketball hoop is often a matter of chance, so let's think of it as a parameter (r).

$$U_{rotational} = P \times r \times \theta = \frac{4\sigma_{fail} I r \theta}{Ly}$$

Equation for rotational energy. P is the force; r is the distance from the point of impact to the center line; θ is the angle the plane rotates before stopping; σ_{fail} is the failure stress; I is the area moment of inertia; L is the rib spacing (i.e. beam length); and y is half the width of the leading edge.

Notice that if the point of contact is near the wing tip (large r), the rotation absorbs lots of energy (so less energy goes into the deformation of the LE). If the point of contact is near the centerline (r is almost 0), the rotation absorbs very little energy, and so more energy goes into deforming the LE. This correlates pretty well with reality. Post-crash inspections of my models have found the inner regions of the leading edge (just outside the prop arc) break more often than out at the tips. Inversely, when one of my models makes a glancing blow, it gets away with minimal damage.

At this point, we have formulae for the maximum energy absorbed by the leading edge before it breaks, and the energy absorbed by the rotation of the model. If all other sources of energy absorption are negligible, then the speed at which the kinetic energy of the model just before impact equals this total energy absorbed during the crash is the critical crash speed that busts the leading edge.

$$KE = U_{spring} + U_{rotational} = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{I\sigma_{fail}^{\ 2}L}{3Ey^2}\right) + \frac{4\sigma_{fail} I r \theta}{Ly}$$

KE is the kinetic energy before impact; U_{spring} is the energy absorbed by the spring (LE); $U_{rotational}$ is the energy absorbed by rotation; m is the mass of the model; v is the speed before impact; I is the area moment of inertia; σ_{fail} is the failure stress; L is the rib spacing (i.e. beam length); E is the Young's Modulus; y is half the width of the leading edge; r is the distance from the point of impact on the LE to the centerline of the plane; and θ is the angle the plane rotates before stopping.

Rearranging this equation and solving for speed gives the following equation for the impact speed that will break the leading edge as a function of known attributes of the LE stick and the plane. This formula can lead us to useful conclusions about how to choose thickness and density for a LE that is less likely to break at a given speed.

$$v = \sqrt{\frac{2}{m}\left[\frac{1}{2}\left(\frac{I\sigma_{fail}^{2}L}{3Ey^{2}}\right) + \frac{4\sigma_{fail}Ir\theta}{Ly}\right]}$$

Equation for critical impact speed that breaks the LE. m is the mass of the model; I is the area moment of inertia; σ_{fail} is the failure stress; L is the rib spacing (i.e. beam length); E is the Young's Modulus; y is half the width of the leading edge; r is the distance from the point of impact to the center line; and θ is the angle the plane rotates before stopping

Enough math. Time for results!

My dime scale Hawker Hurricane weighs about 15 grams, and has a 1/16th inch square, 18 lb/ft^3 leading edge. The rib spacing is about 1.75". The prop diameter is 6", so a worst-case scenario impact location is 3" out from the centerline (just outside the prop arc). The above formula calculates a critical crash speed of 12 miles per hour.[1] Under 12 miles per hour, we would not expect the LE to break when it hits the basketball hoop.

Based on a video taken of my Hurricane at an indoor event, it flies at about 10 miles per hour, so the critical crash speed is just above flight speed. This makes sense based on my observations of that plane. It has crashed into basketball hoops when circling and made out with little damage, but the LE has snapped on the (not infrequent) occasions when the plane hit the hoop rim at a higher speed because it accelerated downward after hitting an obstacle in the rafters.

I have a 16" P-51 that weighs 20 grams with 1.25" rib spacing. I built it with a ⅛" square, 8 lb/ft^3 leading edge, and it has the same 6" prop diameter, so the worst-case impact is still at 3" from the centerline. The formula calculates a critical crash speed of 21 miles per hour, so the leading edge should be safe below that speed.

This also makes sense compared to observations, since my P-51 has suffered almost no leading edge injuries indoors despite flying a little faster than my Hurricane. The extra strength of the LE of my P-51 came with the penalty that the stick weighs twice as much as the LE for the Hurricane.

Now for the question we've all been waiting for: how to make a stronger leading edge without increasing the weight. As noted above, my Hawker Hurricane has an 18 lb/ft^3 1/16" LE. A 3/32"

[1] For a step by step walkthrough of this calculation, see appendix A

square 8 lb/ft^3 stick would weigh exactly the same, as would a 1/16” x ⅛” 9 lb/ft^3 stick. So which is stronger? If we substitute the 3/32” LE in for the 1/16” one in my Hurricane, the critical crash speed increases from 12 mph to 14mph. If we use the 1/16” x ⅛” stick (⅛” dimension running chordwise), the critical speed increases to 16 miles per hour. For the same weight, we’ve made the LE strong enough to withstand a 33% higher crash speed!

The 1/16” x ⅛” stick gets its extra strength from its increased area moment of inertia, and this increase is more than enough to make up for the decreased Young’s Modulus and failure stress that result from its decreased density.

So there you have it, skysters. We now have a scientifically based guideline for choosing the wood for the leading edge that can withstand a higher crash speed with no weight penalty.

Appendix A: Formula Walkthrough

In this appendix, I'll go step by step as I plug in numbers to find the critical crash speed for my Dime Scale Hawker Hurricane. Note: for ease of computation, I'm going to do everything in metric units and convert back to mph at the end.

Here's the final formula:

$$v = \sqrt{\frac{2}{m}\left[\frac{1}{2}\left(\frac{I\sigma_{fail}^{2}L}{3Ey^{2}}\right) + \frac{4\sigma_{fail}Ir\theta}{Ly}\right]}$$

Equation for critical impact speed that breaks the LE. m is the mass of the model; I is the area moment of inertia; σ_{fail} is the failure stress; L is the rib spacing (i.e. beam length); E is the Young's Modulus; y is half the width of the leading edge; r is the distance from the point of impact to the center line; and θ is the angle the plane rotates before stopping

The first thing we're going to need is the mass. My dimer Hurricane weighs about 15 grams, so that means a mass of m = 0.015 kg. While I've got the model in front of me, I'm going to measure the rib spacing, which comes out to L = 1.75" = 0.04445 meters.

The next thing we'll need is the area moment of inertia, I. The LE of my hurricane is a 1/16" (0.001588m) square stick, so we can use the formula for the area moment of inertia of a rectangle. Note that I converted to meters.

$$I = \frac{hb^{3}}{12} = \frac{0.001588 \times 0.001588^{3}}{12} = 5.29 \times 10^{-13}\ [m^{4}]$$

We'll also need the failure stress. In another article, I came up with correlations to calculate failure stress and Young's Modulus based on density. The density of the LE is 18 lb/ft^3.

$$E\,[GPa] = 0.388 \times density\,[lb/ft^{3}] - 1.71 = 0.388 \times 18 - 1.71$$

$$= 5.27\,[GPa] = 5.27 \times 10^{9}[Pa]$$

$$\sigma_{fail}[MPa] = 2.77 \times density\,[lb/ft^{3}] - 4.28 = 2.77 \times 18 - 4.28$$

$$= 45.6\,[MPa] = 4.56 \times 10^{7}[Pa]$$

Since the LE is a 1/16" square stick, y = 0.5 * 1/16" = 1/32" = 0.00079375 meters.

The prop diameter is 6", so a worst-case scenario impact location is 3" out from the centerline (just outside the prop arc), so let's use r = 3" = 0.0762 meters.

Based on my memory of indoor ships crashing, the angle of rotation seems often to be about 60 degrees. The angle, θ, has to be in radians, and 60 degrees is almost 1 radian, so I'll use θ = 1 radian.

That's all the numbers gathered. Time to plug in:

$$v = \sqrt{\frac{2}{0.015}\left[\frac{1}{2}\left(\frac{5.29\times10^{-13}\times(4.56\times10^{7})^{2}\times0.04445}{3\times5.27\times10^{9}\times0.00079375^{2}}\right) + \frac{4\times4.56\times10^{7}\times5.29\times10^{-13}\times0.0762\times1}{0.04445\times0.00079375}\right]} = 5.3\,\frac{m}{s} = 12\,mph$$